MODIFICATION TO THE SHADING ROUTINE IN THE HREC ORBITAL DRAG COEFFICIENT COMPUTER PROGRAM

FINAL REPORT

September 1968

MISSILES & SPACE COMPANY

A GROUP DIVISION OF LOCKHEED AIRCRAFT CORPORATION

SUNNYVALE, CALIFORNIA

HREC-0230-1
LMSC/HREC A791832

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MODIFICATION TO THE SHADING ROUTINE IN THE HREC ORBITAL DRAG COEFFICIENT COMPUTER PROGRAM

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Contract NAS8-20230
Mod. 2., DCN 17-75-20109

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FOREWORD

This document presents the work accomplished under Contract NAS8-20230, Modification 2, DCN 17-75-20109, "Study of Drag Coefficients for Unusual Vehicle Configurations, Modification 2." The work was performed by the Thermal Environment Section of Lockheed's Huntsville Research & Engineering Center for the Aero-Astrodynamic Laboratory of Marshall Space Flight Center.

The NASA Technical Supervisors for this study were Messrs. J. O. Ballance and J. D. Johnson of R-AERO-AE, and R. F. Elkin of R-AERO-AT.

SUMMARY

Lockheed's Huntsville Research & Engineering Center developed a computer program which determines aerodynamic coefficients for complex vehicle configurations. Modifications were made which are related to determining the shading effects on the aerodynamic coefficients and the type of surfaces available for input.

The shading technique consists of dividing each surface on the vehicle into small incremental areas, each of which is checked to determine if the oncoming flow, represented by a velocity vector, intersects another surface on the vehicle before it reaches the incremental area being checked (i.e., is the incremental area shaded?). If the incremental area is not shaded its contribution to the aerodynamic coefficients for the surface is calculated.

When the vehicle is represented by a large number of surfaces and incremental areas, the computer time required to determine the shading effects can be excessive. A pre-shade routine was developed and incorporated into the program to determine all those surfaces which cannot shade any portion of the surface under consideration. The "no shade" surfaces are then excluded from the checks made on the incremental areas. Computer time saved by this routine can be as much as 30% for some problem types.

A description of the shading and pre-shade techniques is included in this report as well as an input guide for the modified computer program and a description of the program's output. A sample problem is included with a listing of the input data and portions of the output.

CONTENTS

ILLUSTRATIONS

NOMENCLATURE

a, b, c	coefficients of the equation for the conic shape
C_1, C_2, C_3	coefficients defined by set of Equations (25)
$\bar{i}_i, \bar{j}_i, \bar{k}_i$	unit vectors in the individual body coordinate system
$\bar{i}_c, \bar{j}_c, \bar{k}_c$	unit vectors in the composite body coordinate system
$\left.\begin{matrix} \ell_{xt}, \ell_{yt}, \ell_{zt} \\ m_{xt}, m_{yt}, m_{zt} \\ n_{xt}, n_{yt}, n_{zt} \end{matrix}\right\}$	direction cosines between the test cylinder and composite coordinate system as defined by set of Equations (31), (32) and (33), respectively
$\bar{n}_i$	unit vector normal to elemental area
$\bar{P}$	vector location of a point on a surface in composite coordinates
R_1, R_2, R_3	coefficients defined by set of Equations (24)
r_i	magnitude of radius measured from the centerline of the body to the elemental area
$\bar{R}_i$	vector location of a point in individual coordinate system
$\bar{R}_{c2i}$	vector from origin of composite system to the origin of the individual system
∇S_i	body surface gradient
$\left[T_{i2c}\right]$	matrix to transform from individual to composite coordinate systems
$\left[T_{c2i'}\right]$	matrix to transform from composite to individual coordinate system
$\bar{V}_c$	unit vector in the direction of molecular velocity

Nomenclature (Continued)

Greek

α_o	vehicle angle of attack
$\alpha, \beta_{min}, \beta_{max}$, $\gamma_{min}, \gamma_{max}$	dimensions defined for each type of surface as shown in Figure 1
β_o	vehicle angle of roll
ζ, η, ρ	coefficients defined by Equations (21), (22) and (23), respectively
λ	scale factor used to determine whether or not an element is shaded
φ_o	angle of integration around conic shape
ϕ, ψ, ω	Eulerian angles between the individual and composite coordinate system as shown in Figure 2.

Section 1
INTRODUCTION

Lockheed Missiles & Space Company, Huntsville Research & Engineering Center (Lockheed/Huntsville) conducted a study to determine the aerodynamic characteristics of orbital vehicles in the continuum, transition and free-molecular flow regimes. As a part of this study, a large computer program was developed to be used to determine the force and moment coefficients for any vehicle or vehicle cluster at any angle of attack, roll or combination of both. Within this program, vehicle clusters are analyzed by treating each composite part of the shape as a separate body, accounting for interface reactions and mutual molecular shadowing, and summing the separate results. Aerodynamic forces are computed in the free molecule flow regime using diffuse free molecule theory and in the continuum flow regime using modified Newtonian theory. For the transition flow regime, an empirical relation was used which was developed to approximate experimental data (see Reference 1).

Under Modification 2 to this contract, Lockheed/Huntsville developed a version of the computer program mentioned above. Modifications to this program consisted of: (1) increasing the number of types of shapes available; (2) modifying the existing shading techniques to make it applicable to new types of shapes; (3) increasing program accuracy; and (4) adding a preshade subroutine which will scan all surfaces and eliminate from further checking those which cannot shade any portion of the surface under consideration in order to reduce computer run time.

Shading techniques used in the modified computer program are documented. Input data requirements and program output are described and a sample problem presented.

Reference 1 contains a description of the techniques used in calculating the aerodynamic force and moment coefficients.

Section 2
DESCRIPTION OF SURFACE TYPES (GEOMETRY)

2.1 TYPES OF SURFACES

The program can evaluate shading for the following surface types:

- Rectangular plates
- Circular plates
- Trapezoidal plates
- Cylinders
- Cone or cone frustums
- Spheres*
- Circular paraboloids*

The types of surfaces and their dimensions in the individual coordinate systems are shown in Figure 1, pages 3, 4 and 5.

2.2 COORDINATE SYSTEMS

Complex vehicle and vehicle clusters are resolved into a series of surfaces of the types mentioned above. Each surface is described in its own coordinate system which is designated as "individual coordinate system." The reference system of the vehicle or vehicle cluster to which all individual coordinate systems are referenced, is called the central coordinate system. Since the individual coordinate system is fixed in space and time with respect to the central coordinate system, a transformation exists, which is invariant with time, that relates the surface to the central coordinate system.

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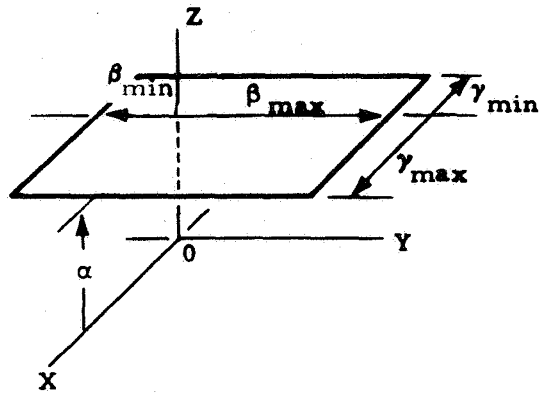
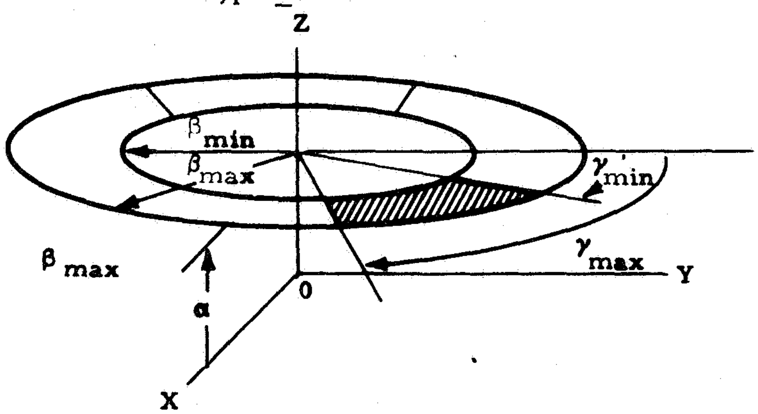
*The computer program presently does not have the capability for calculating the aerodynamic force and moment coefficients for spheres or circular paraboloids.

Surface Type ± 1 Rectangle



$\beta_{min} < \beta_{max}$
$\gamma_{min} < \gamma_{max}$

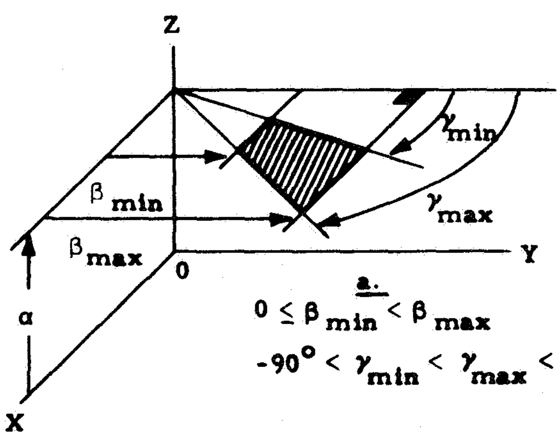
Surface Type ± 2 Disc



$0 \leq \beta_{min} < \beta_{max}$

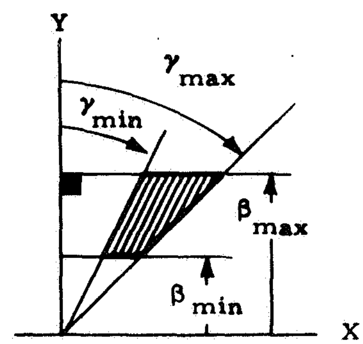
$$\begin{cases} 0^\circ < \gamma_{min} < \gamma_{max} \leq +360^\circ \\ \gamma_{max} \leq \gamma_{min} + 360^\circ \end{cases}$$

Surface Type ± 3 Trapezoid



a.

$0 \leq \beta_{min} < \beta_{max}$
$-90^\circ < \gamma_{min} < \gamma_{max} < +90^\circ$

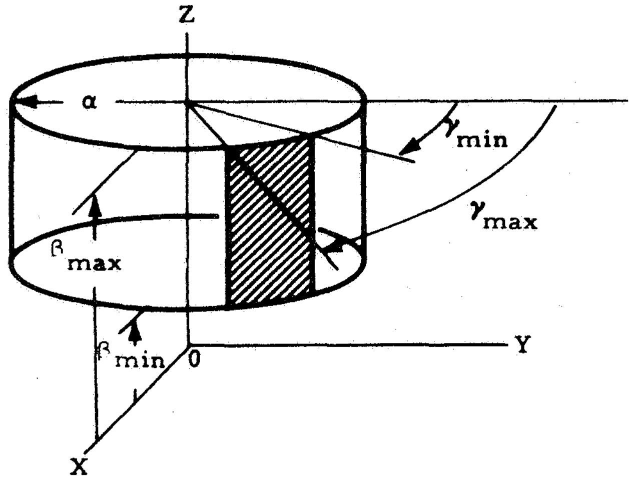


b.

$\beta_{min} < \beta_{max} \leq 0$
$+90^\circ < \gamma_{min} < \gamma_{max} < +270^\circ$

Figure 1 – Input Surface Types (Sheet 1)

Surface Type ± 4 Cylinder

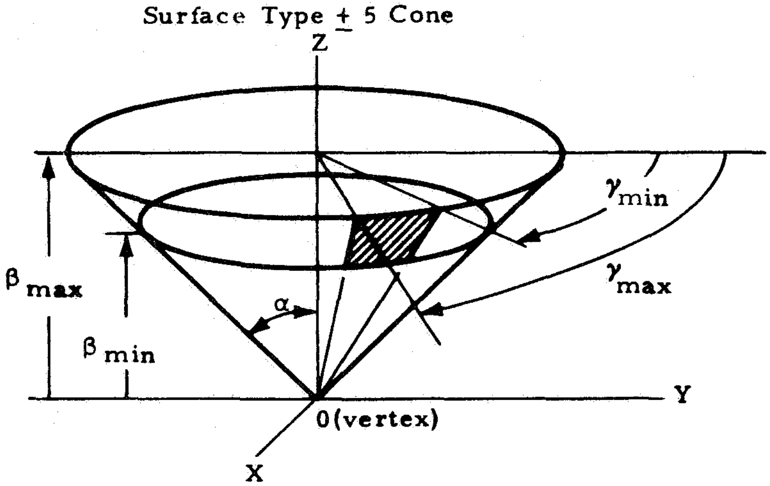


$0 < \alpha$

$\beta_{min} < \beta_{max}$

$$\begin{cases} 0^\circ < \gamma_{min} < \gamma_{max} \le +360^\circ \\ \gamma_{max} \le \gamma_{min} + 360^\circ \end{cases}$$

Surface Type ± 5 Cone

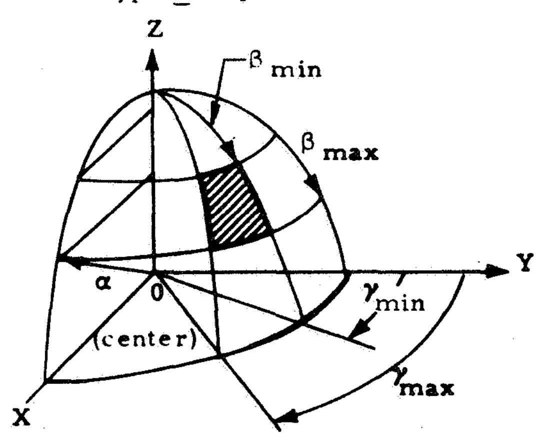


$0 < \alpha$

$0 \le \beta_{min} < \beta_{max}$

$$\begin{cases} 0^\circ < \gamma_{min} < \gamma_{max} \le +360^\circ \\ \gamma_{max} \le \gamma_{min} + 360^\circ \end{cases}$$

Surface Type ± 6 Sphere



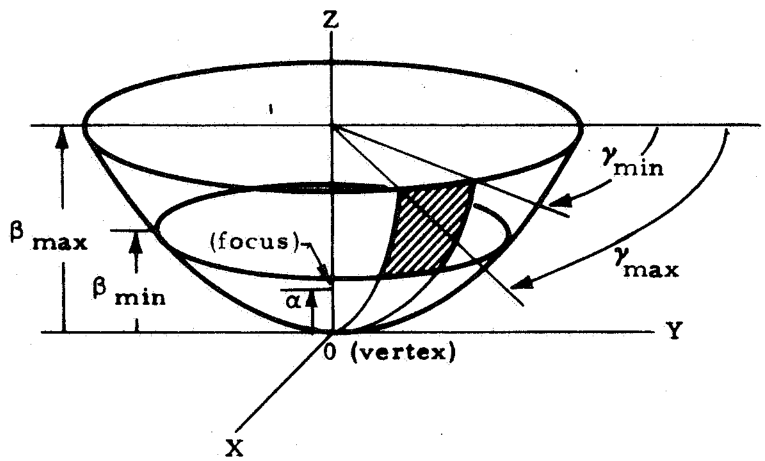
$0 < \alpha$

$0 \le \beta_{min} < \beta_{max} \quad 180^\circ$

$$\begin{cases} 0^\circ < \gamma_{min} < \gamma_{max} \le +360^\circ \\ \gamma_{max} \le \gamma_{min} + 360^\circ \end{cases}$$

Figure 1 – Input Surface Types (Sheet 2)

Surface Type ± 7 Circular Paraboloid



$0 < \alpha$

$0 \leq \beta_{min} < \beta_{max}$

$$\begin{cases} 0^{\circ} < \gamma_{min} < \gamma_{max} \leq +360^{\circ} \\ \gamma_{max} \leq \gamma_{min} + 360^{\circ} \end{cases}$$

Figure 1 – Input Surface Types (Sheet 3)

The relation between the individual coordinate and the central coordinate system is given by the following transformation matrix and translational vector defined as

$$\left[T_{i2c}\right] \equiv \begin{bmatrix} \ell_{xi} & m_{xi} & n_{xi} \\ \ell_{yi} & m_{yi} & n_{yi} \\ \ell_{zi} & m_{zi} & n_{zi} \end{bmatrix} \tag{1}$$

$$\vec{R}_{c2i} \equiv R_x \vec{i}_c + R_y \vec{j}_c + R_z \vec{k}_c \tag{2}$$

where

$$\left.\begin{aligned} \ell_{xi} &= \cos\phi \cos\psi \\ m_{xi} &= -\sin\phi \cos\psi \\ n_{xi} &= \sin\psi \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} \ell_{yi} &= \sin\phi \cos\omega - \cos\phi \sin\psi \sin\omega \\ m_{yi} &= \cos\phi \cos\omega + \sin\phi \sin\psi \sin\omega \\ n_{yi} &= \cos\psi \sin\omega \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} \ell_{zi} &= -\sin\phi \sin\omega - \cos\phi \sin\psi \cos\omega \\ m_{zi} &= -\cos\phi \sin\omega + \sin\psi \sin\phi \cos\omega \\ n_{zi} &= \cos\psi \cos\omega \end{aligned}\right\} \tag{5}$$

and where R_x, R_y, R_z, ϕ, ψ and ω are given in Figure 2.

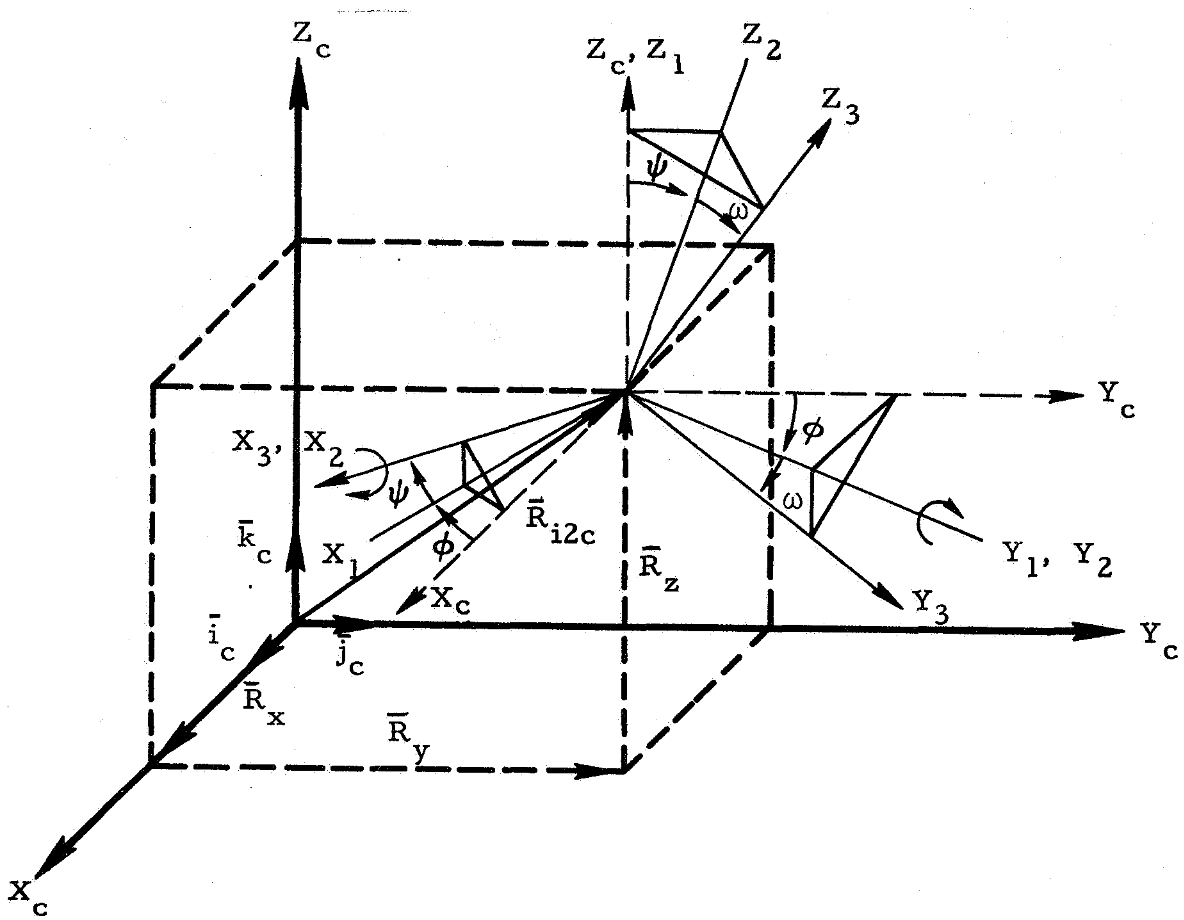


Figure 2 – Relative Position of Individual Coordinate System with Respect to Central Coordinate System

The orientation of each individual coordinate system is specified by three angles ϕ, ψ and ω. These angles define three coordinate rotations that rotate the central coordinates (X_c, Y_c, Z_c) into the surface coordinates (X_3, Y_3, Z_3). The rotations are in the direction $X_c, Y_c, Z_c \longrightarrow X_3, Y_3, Z_3$ and must be carried out in the strict order ϕ, then ψ, then ω, as follows:

ϕ rotation: Rotate Y_c toward X_c about Z_c obtaining X_1, Y_1, Z_1 system

ψ rotation: Rotate X_1 toward Z_1 about Y_1 obtaining X_2, Y_2, Z_2 system

ω rotation: Rotate Y_2 toward Z_2 about X_2 obtaining X_3, Y_3, Z_3 system.

2.3 FLAT SURFACES

The three flat surfaces, rectangular plate, circular plate and trapezoidal plate, are described by a unit vector normal to the surface in the individual coordinate system and by the "β_{min}", "β_{max}" and "γ_{min}", "γ_{max}" limitations described in Figure 1.

The normal to the surface is given by

$$\vec{n}_i = \pm\vec{k}_i = \pm \left\{ \vec{i}_i \times \vec{j}_i \right\} \tag{6}$$

and a point $\overline{P}$ on the surface given in individual coordinate system is transformed to the central coordinate system by the following relation

$$\overline{P} = \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \left[T_{i2c}\right] \begin{bmatrix} X_i \\ Y_i \\ \alpha \end{bmatrix} + \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix} . \tag{7}$$

2.4 CONIC SURFACES

The conic surfaces consist of the four remaining surfaces, the cylinder, the cone or cone frustum, the sphere and the circular paraboloid.

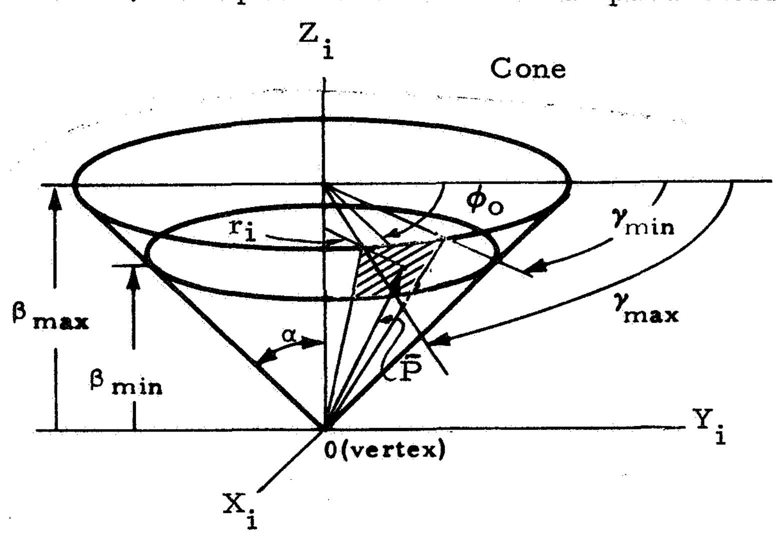


Figure 3 – Example of Conic Surface

Any point, $\overline{P}$, in the individual coordinate system is described by the vector equation

$$\overline{P} = X_i \vec{i}_i + Y_i \vec{j}_i + Z_i \vec{k}_i \tag{8}$$

A point constrained to lie on the surface will satisfy the relation

$$S_i (X_i, Y_i, Z_i) = 0 \tag{9}$$

and where

$$r_i = (X_i^2 + Y_i^2)^{1/2} \tag{10}$$

and

$$X_i = r_i \sin\phi_o \tag{11}$$

$$Y_i = r_i \cos\phi_o \quad . \tag{12}$$

All conic surfaces can be expressed mathematically by

$$r_i - (a + b Z_i + c Z_i^2)^{1/2} = 0$$

for values of "Z_i" for the cylinder, cone or cone frustum and circular paraboloid given by

$$\beta_{min} \leq Z_i \leq \beta_{max}$$

and for the sphere given by

$$\alpha \cos(\beta_{min}) \leq Z_i \leq \alpha \cos(\beta_{max})$$

The constants a, b and c take on the following values depending on the type of surface:

$$\text{cylinder} \begin{cases} a = \alpha^2 \\ b = 0 \\ c = 0 \end{cases}$$

$$\text{cone or cone frustum} \begin{cases} a = 0 \\ b = 0 \\ c = \tan^2 \alpha \end{cases}$$

$$\text{sphere} \begin{cases} a = \alpha^2 \\ b = 0 \\ c = -1 \end{cases}$$

$$\text{circular paraboloid} \begin{cases} a = 0 \\ b = 4\,\alpha \\ c = 0 \end{cases}$$

A unit vector normal to the surface at a point $\overline{P}$, is defined by

$$\overline{n} \equiv \frac{\nabla S_i}{|\nabla S_i|} = \frac{\left(\frac{\partial}{\partial X_i} S_i \, \vec{i}_i + \frac{\partial}{\partial Y_i} S_i \, \vec{j}_i + \frac{\partial}{\partial Z_i} S_i \, \vec{k}_i \right)}{|\nabla S_i|}$$

where

$$|\nabla S_i| \equiv \left[\left(\frac{\partial S_i}{\partial X_i} \right)^2 + \left(\frac{\partial S_i}{\partial Y_i} \right)^2 + \left(\frac{\partial S_i}{\partial Z_i} \right)^2 \right]^{1/2}$$

which for the conic equation reduces to

$$\bar{n} = \frac{\sin\phi_o\ \vec{i}_i + \cos\phi_o\ \vec{j}_i + \left[- \dfrac{(b + 2\ c\ Z_i)}{2\ r_i} \right] \vec{k}_i}{|\nabla S_i|} \tag{13}$$

where

$$|\nabla S_i| = \sqrt{\left(\frac{b + 2\ c\ Z_i}{2\ r_i}\right)^2 + 1} \quad .$$

Similar to the flat surfaces, a point, P, can be given in the central coordinate system as

$$\bar{P} = \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} T_{i2c} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} + \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix} \tag{14}$$

2.5 METHOD OF INTEGRATION

The method of integration of each surface is done as follows: (1) the surface is divided into an arbitrary number of elemental areas; (2) the center of area of the elemental area in the individual coordinate system is determined; and (3) the surface normal at this point, assuming the absolute value of the elemental area to be concentrated at the center of area, is evaluated. The integration of the conic surfaces takes place along the "Z_i" direction between the limits of "β_{min}" and "β_{max}" and around the body between the limit of "γ_{min}" and "γ_{max}" in arbitrary step sizes. In the case of a rectangular flat plate, integration takes place in rectangular strips, while in the case of a circular plate, integration takes place in circular strips and for a trapezoidal plate, the integration is performed by summing trapezoidal elements, the integration being performed between the limits of "β_{min}" and "β_{max}," and "γ_{min}" and "γ_{max}."

The arbitrary integration step size is left to the discretion of the user. The smaller the elemental areas, the more accurate the integration process and the longer the computer run time.

Section 3
SHADOWING

To calculate the forces on a given element, it is necessary to determine whether the elemental area presents itself to the flow, or whether the velocity flow vector intersects any other surface before it intersects the elemental area. In other words, whether or not the element is shaded must be determined.

An elemental area can be shaded in two ways:

- The elemental area is located on the back node of a body (opposite from the flow direction), or
- One or more surfaces are between the velocity flow vector and the elemental area.

The first case is easily checked. Since the normal to the elemental area has been calculated in the individual coordinate system, and since the velocity flow vector is given in the central coordinate system, a dot product is sufficient to determine if the elemental area does not present itself to the flow, (see Figure 4 on next page). If the elemental area does not present itself to the flow then,

$$\vec{V}_c \cdot \left\{ \left[T_{i2c} \right] \right\} \vec{n}_i \quad > \quad 0 \tag{15}$$

That is, if the angle between the velocity vector and the normal vector is between $+90^{\circ}$ and -90°, the centroid of the element is on the surface side away from the flow. If the above test shows that the element is shadowed, no further test is necessary and forces can be calculated accordingly. However, if the test shows that the element presents itself to the flow, additional checks must be made to determine if the element is shadowed by another surface.

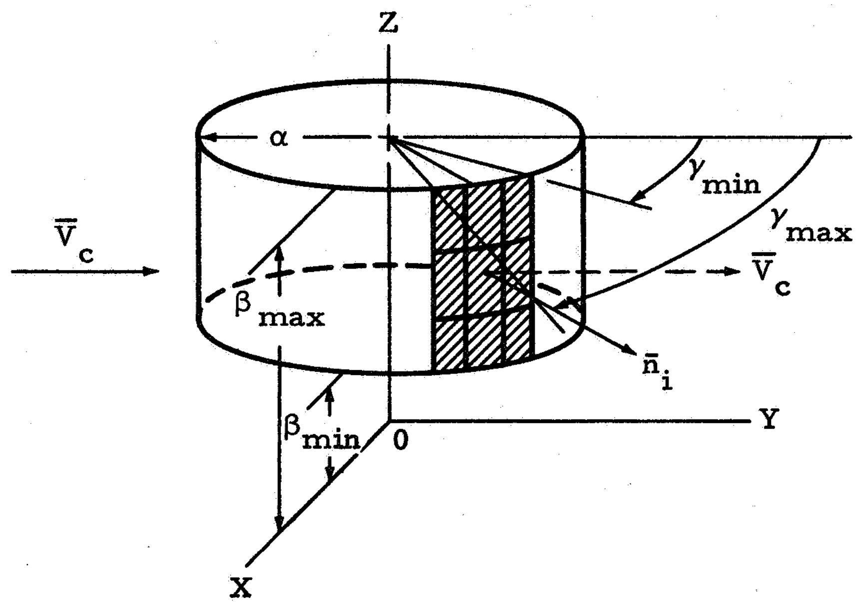


Figure 4 – Example of Dot Product

This check is somewhat more complex than the preceding check; however, it is still a straightforward vector operation. The method is discussed in the following paragraphs.

In Figure 5 an elemental area in the "i" system is set up by the integration routine. It is desired to know whether the elemental area is shadowed by some other surface from the given velocity flow vector.

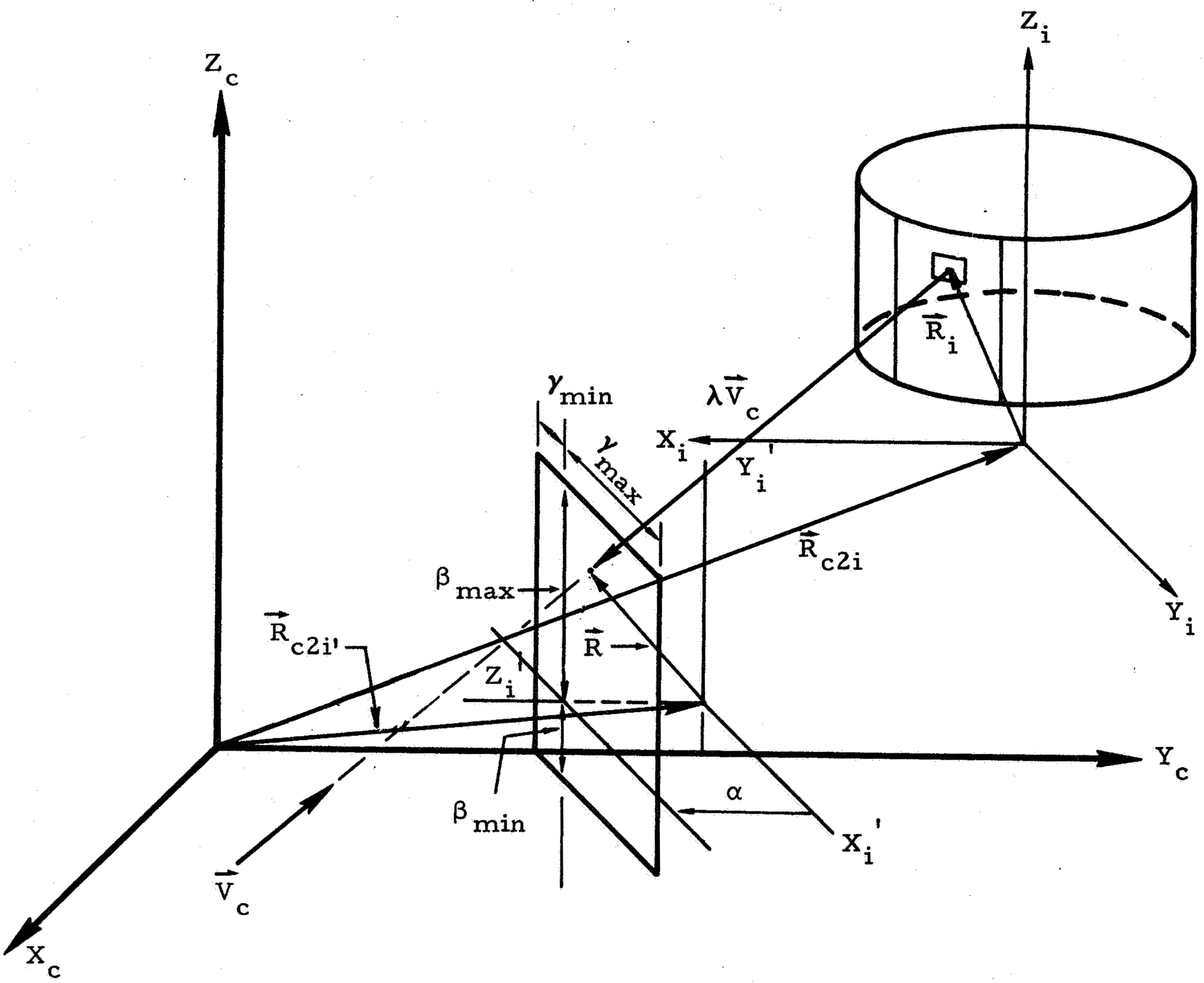


Figure 5 - Vector Operation Diagram for Shading Check

From the figure it can be seen that the vector $\vec{R}$ is given by

$$\vec{R} = \left[T_{c2i'}\right] \left\{ \vec{R}_{c2i} + \left[T_{i2c}\right] \vec{R}_i + \lambda\vec{V} - \vec{R}_{c2i'} \right\} \tag{16}$$

where

$$\left[T_{c2i}{}'\right] \equiv \begin{bmatrix} \ell_{xi}{}' & \ell_{yi}{}' & \ell_{zi}{}' \\ m_{xi}{}' & m_{yi}{}' & m_{zi}{}' \\ n_{xi}{}' & n_{yi}{}' & n_{zi}{}' \end{bmatrix} \tag{17}$$

$$\vec{V}_c \equiv V_x \vec{i}_c + V_y \vec{j}_c + V_z \vec{k}_c \tag{18}$$

$$\vec{R}_{c2i}{}' \equiv R_x{}' \vec{i}_c + R_y{}' \vec{j}_c + R_z{}' \vec{k}_c \tag{19}$$

and where "λ" is a scale factor, which is the variable to be determined. It can then be said that if there exists a value of λ which will produce a point in the "i'" system which satisfies the equation of the surface in the "i'" system, the elemental area is shaded by this surface. If no real value exists then the element is not shaded by the i' surface.

By vector manipulation, the following vector equation in the unknown "λ" is obtained:

$$\vec{R} = \left[R_1 + \lambda C_1\right] \vec{i}' + \left[R_2 + \lambda C_2\right] \vec{j}' + \left[R_3 + \lambda C_3\right] \vec{k}' \tag{20}$$

where if we define

$$\zeta \equiv R_x - R_x{}' + \ell_{xi} X_i + m_{xi} Y_i + n_{xi} Z_i \tag{21}$$

$$\eta \equiv R_y - R_y{}' + \ell_{yi} X_i + m_{yi} Y_i + n_{yi} Z_i \tag{22}$$

$$\rho \equiv R_z - R_z{}' + \ell_{zi} X_i + m_{zi} Y_i + n_{zi} Z_i \tag{23}$$

then the constants R_1, R_2, R_3, C_1, C_2, C_3 can be represented as

$$\left.\begin{aligned} R_1 &= \ell_{xi}'\ \xi + \ell_{yi}'\ \eta + \ell_{zi}'\ \rho \\ R_2 &= m_{xi}'\ \xi + m_{yi}'\ \eta + m_{zi}'\ \rho \\ R_3 &= n_{xi}'\ \xi + n_{yi}'\ \eta + n_{zi}'\ \rho \end{aligned}\right\} \tag{24}$$

$$\left.\begin{aligned} C_1 &= \ell_{xi}'\ V_x + \ell_{yi}'\ V_y + \ell_{zi}'\ V_z \\ C_2 &= m_{xi}'\ V_x + m_{yi}'\ V_y + m_{zi}'\ V_z \\ C_3 &= n_{xi}'\ V_x + n_{yi}'\ V_y + n_{zi}'\ V_z \end{aligned}\right\} \tag{25}$$

The solution of the vector equation for the unknown "λ" is very simple when the intersecting surface is a flat plate.

If the vector is to intersect a flat surface, then the following equation has to be satisfied

$$R_3 + \lambda C_3 = \alpha$$

for which "λ" can be solved

$$\lambda = (\alpha - R_3)/C_3 \tag{26}$$

With this value of "λ" the i' and j' components of $\overrightarrow{R}$ can be evaluated and tested to see whether they lie within the limits of the plate, i.e., for a rectangular plate

$$\gamma_{min} \leq R_1 + \lambda C_1 \leq \gamma_{max}$$

$$\beta_{min} \leq R_2 + \lambda C_2 \leq \beta_{max}$$

in order for it to intersect vector $\vec{R}$ and cause shading on the element of the surface being tested.

If the intersecting surface is a conic, the solution of the vector equation for the unknown "λ" is obtained by substituting the i', j' and k' components of $\vec{R}$ into the general conic equation

$$r = (x'^2 + y'^2)^{1/2} = (a + b\, z' + c\, z'^2)^{1/2}$$

This substitution will yield a quadratic equation in "λ" and the solution is given by

$$\lambda = \frac{-(2 R_1 C_1 + 2 R_2 C_2 - b C_3) \pm \left\{ (2 R_1 C_1 + 2 R_2 C_2 - b C_3)^2 + 4 (C_1^2 + C_2^2 - c\, C_3^2) \left[R_1^2 + R_2^2 - a + (b+c) R_3 \right] \right\}^{1/2}}{2 (C_1^2 + C_2^2 - c\, C_3^2)} \qquad (27)$$

If the values of "λ" are real and negative there is a possibility of two points of intersection. If the roots are real, negative and equal there is a possibility of one point of intersection. If the roots are either real, positive or imaginary or if the denominator

$$C_1^2 + C_2^2 - c\, C_3^2 = 0 \qquad (28)$$

no intersection is possible.

If the intersection is possible then the i', j' and k' components of $\vec{R}$ are evaluated with the "λ" values obtained and tested to see whether they lie within the limits of the conic surface.

Section 4
PRESHADE

Since the Aerodynamic Coefficient Computer Program scanned all surfaces for checking the shading on a given surface, and did not eliminate from further checking those surfaces which cannot shade any portion of the surface under consideration, a preshade routine was developed. An input flag is used first to indicate if a given surface can be shaded and if it can shade other surfaces for each given flow. The preshade routine flags the surfaces that have been input as "can shade" if they cannot shade any portion of the surface for which the aerodynamic forces are being calculated. The flagged surfaces are eliminated from the more complex shading checks which are made on each element, thus reducing the required computer time.

The preshade routine consists of constructing a cylinder around the surface that can be shaded with a radius that will enclose the whole surface. The ends of this cylinder will be perpendicular to the flow and the length will be from the origin of each individual coordinate system of the surface being tested to the outermost surface presented to the oncoming flow. The radius of the test cylinder depends on the type of surface that it encloses.

The free molecular flow is described by an angle of attack, α_o and angle of roll, β_o. The unit velocity vector has three components in the X_c-Y_c-Z_c directions, respectively, given by

$$\begin{aligned} V_x &= \sin\alpha_o \cos\beta_o \\ V_y &= \sin\alpha_o \sin\beta_o \\ V_z &= \cos\alpha_o \end{aligned} \tag{30}$$

The unit normal to the ends of the cylinder is given by the negative of the unit flow vector.

The test cylinder coordinate system is then described with respect to the central coordinate system by defining its direction cosines as a function of the angle of attack, α_o, and the angle of roll, β_o.

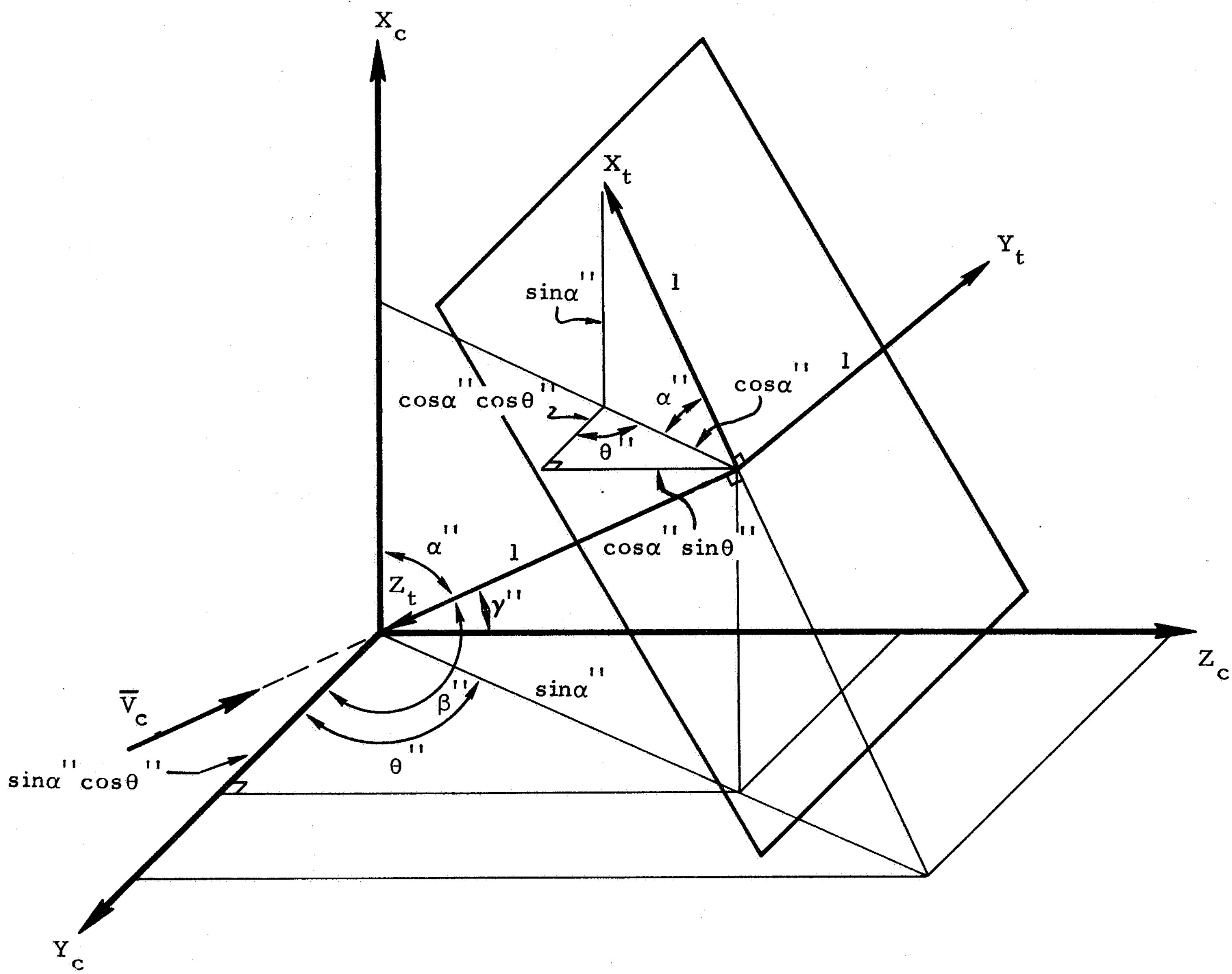


Figure 6 – Relation Between Test Cylinder Coordinate System and Central Coordinate System

By substituting the relations of the angle of attack "α_o" and roll "β_o" into the trigonometric relations derived from the figure, the direction cosines, (see Figure 5, previous page) were found to be

$$\left.\begin{aligned}
l_{xt} &= (1 - \sin^2\alpha_o \cos^2\beta_o)^{1/2} \\
l_{yt} &= -\sin^2\alpha_o \sin\beta_o \cos\beta_o/(1 - \sin^2\alpha_o \cos^2\beta_o)^{1/2} \\
l_{zt} &= -\sin\alpha_o \cos\alpha_o \cos\beta_o/(1 - \sin^2\alpha_o \cos^2\beta_o)^{1/2}
\end{aligned}\right\} \quad (31)$$

$$\left.\begin{aligned}
m_{xt} &= 0 \\
m_{yt} &= -\cos\alpha_o/(1 - \sin^2\alpha_o \cos^2\beta_o)^{1/2} \\
m_{zt} &= \sin\alpha_o \sin\beta_o/(1 - \sin^2\alpha_o \cos^2\beta_o)^{1/2}
\end{aligned}\right\} \quad (32)$$

$$\left.\begin{aligned}
n_{xt} &= -\sin\alpha_o \cos\beta_o \\
n_{yt} &= -\sin\alpha_o \sin\beta_o \\
n_{zt} &= -\cos\alpha_o
\end{aligned}\right\} \quad (33)$$

The above direction cosines are not valid for the following combination of angles of attack and roll

$$\alpha_o = 90^o, \quad \beta_o = 0^o$$

$$\alpha_o = 90^o, \quad \beta_o = 180^o$$

$$\alpha_o = 270^o, \quad \beta_o = 0^o$$

$$\alpha_o = 270^o, \quad \beta_o = 180^o$$

for which the direction cosines become trivial. The direction cosines define the transformation matrix between the central coordinate system and test cylinder system.

After the test cylinder is defined, a circle is constructed around the surface that can cause shading and is transformed to the test cylinder coordinate system such that the circle lies parallel to the X_t-Y_t plane. The circle is constructed around the surface and on the extreme point of the surface which presents itself first to the oncoming flow. The point is tested to see if its axial component lies in the positive axial direction of the test cylinder coordinate system. If it does, then the sum of the radius of the test cylinder and the radius of the cylinder around the surface that can shade has to be greater than the distance between the origins of these two cylinders in the plane perpendicular to the flow (Figure 6) in order to cause shading on the surface being tested for that particular combination of angles of attack and roll.

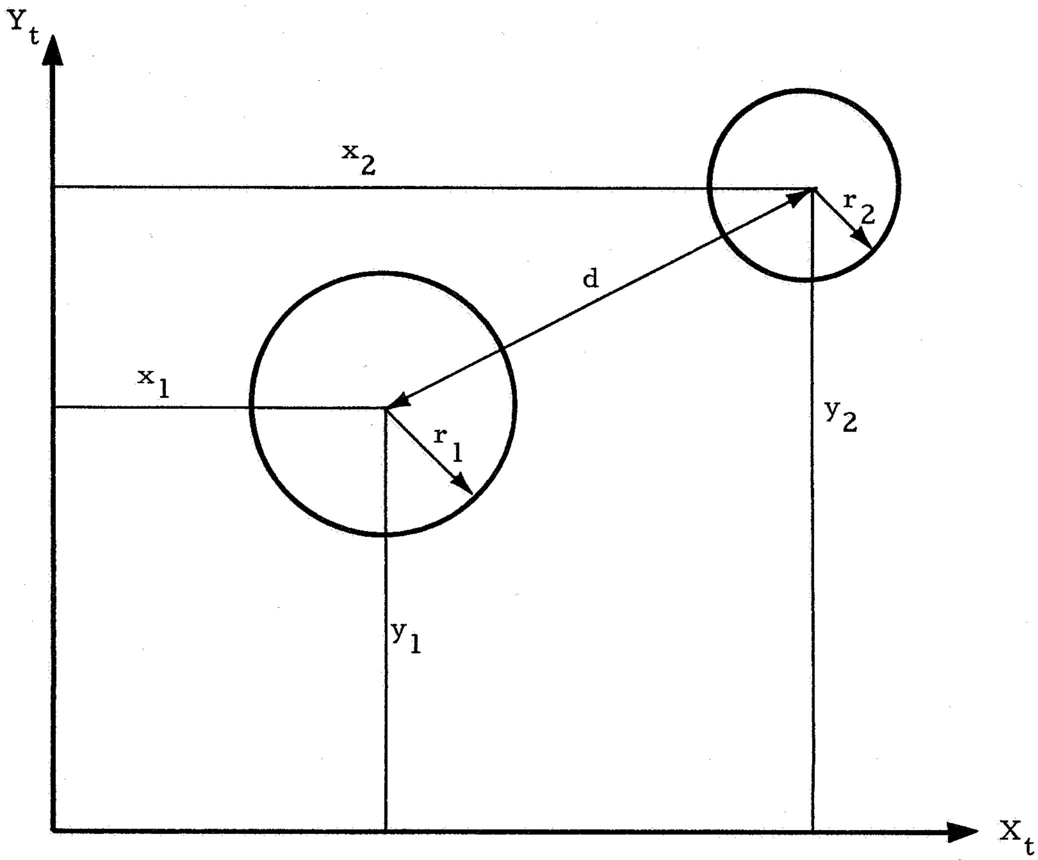
It can be noted from Figure 6 that

$r_1 + r_2 < d$ circles are outside and have no point in common

$r_1 + r_2 = d$ circles are outside and have one point in common

$r_1 + r_2 > d$ circles are either partially or totally inside

The last condition indicates that the surface has a possibility of shading the surface around which the test cylinder was constructed.



$$d = \left\{ (x_2 - x_1)^2 + (y_2 - y_1)^2 \right\}^{1/2}$$

Figure 7 - Preshade Test Cylinders (flow perpendicular to the X_tY_t plane)

Section 5
PROGRAM DESCRIPTION

5.1 INPUT PROCEDURE

Limitations are presently placed on the inputs in the modified version of the program described in this report. The following inputs are allowable:

- Fifty (50) separate problems in one program run
- One-hundred (100) maximum subshapes per vehicle cluster
- One-hundred (100) combinations of angle of attack and roll
- Ten (10) maximum altitudes input with each problem
- Unlimited amount of elements on each subshape (Limited only by computer run time).

No specific unit is required for the inputs, however all dimensions used must be consistent with type of unit selected.

The inputs to the program are:

Card	Columns	Format	Variable Name	Description
1	1 - 72	12A6	COMMNT	A comment used for problem identification.
2	1 - 5	I5	NUMBER	Number of subshapes.
	6 - 10	I5	IANGLE	Number of combinations of angle of attack and roll.

Card	Columns	Format	Variable Name	Description
	11 - 15	I5	NOALTS	Number of altitudes
	16 - 25	F10.5	RREF	Reference radius for the area on which the coefficients are based
	26 - 35	F10.5	XI	Length of the vehicle
	36 - 45	F10.5	DI	Characteristic diameter of the vehicle
3*	21 - 30	F10.5	SR(JET)	Molecular speed ratio
	41 - 50	F10.5	TEMRAT(JET)	Kinetic temperature ratio
	61 - 80	F20.9	ALAMDA(JET)	Mean free path of the molecules
4**	1 - 5	I5	ISURFN(I)	Surface identification number. This number is printed out with the data and is used to identify surfaces with a vehicle drawing
	6 - 10	I5	IDSURF(I)	Identify the type of subshape, i.e., ±1 = rectangular plate ±2 = circular plate ±3 = trapezoidal plate ±4 = cylinder ±5 = cone or cone frustum ±6 = sphere*** ±7 = circular paraboloid*** NOTE: + sign for outside surface - sign for inside surface
	11 - 15	I5	NVB(I)	Number of element divisions along the described Beta direction of the surface, between the Beta limits according to the type of surface (see Figure 1)

*One card for each altitude.

**One card for each surface.

***These surface types are currently not available.

Card	Columns	Format	Variable Name	Description
	16 - 20	I5	NVG(I)	Number of element divisions along the described Gamma direction of the surface, between the Gamma limits according to the type of surface (see Figure 1)
	21 - 25	I5	NB(I)	Presently not used
	26 - 30	I5	NG(I)	Presently not used
	31 - 40	E10.6	A(I)	Value of "α" according to the type of surface (see Figure 1)
	41 - 50	E10.6	BMIN(I)	Lower limit in the Beta direction according to the type of surface (see Figure 1)
	51 - 60	E10.6	BMAX(I)	Upper limit in the Beta direction according to the type of surface (see Figure 1)
	61 - 70	E10.6	GMIN(I)	Lower limit in the Gamma direction according to the type of surface (see Figure 1)
	71 - 80	E10.6	GMAX(I)	Upper limit in the Gamma direction according to the type of surface (see Figure 1)
5*	21 - 30	E10.6	RX(I)	The X_c location of the individual coordinate system origin in the central coordinate system (see Figure 2)
	31 - 40	E10.6	RY(I)	The Y_c location of the individual coordinate system origin in the central coordinate system (see Figure 2)
	41 - 50	E10.6	RZ(I)	The Z_c location of the individual coordinate system origin in the central coordinate system (see Figure 2)

*One card for each surface.

Card	Columns	Format	Variable Name	Description
	51 - 60	E10.6	PHI(I)	The "ϕ" angle between the individual and central coordinate system as defined in Figure 2
	61 - 70	E10.6	PSI(I)	The "ψ" angle between the individual and central coordinate system as defined in Figure 2
	71 - 80	E10.6	OMEGA(I)	The "ω" angle between the individual and central coordinate system as defined in Figure 2
6*	1 - 20	E20.8	ALPHA(I)	Angle of attack in the central coordinate
	21 - 40	E20.8	BETA(I)	Angle of roll in the central coordinate
	41 - 45	I5	NOSCS(I)	Number of surfaces that can be shaded for the given angle of attack and roll
7**	1-2, 5-6, 9-10	I2	NCBS(ISURF)	Shading flag for each surface 1 = can be shaded 0 = cannot be shaded
	3-4, 7-8, 11-12	I2	NCS(ISURF)	Shading flag for each surface 1 = can shade 0 = cannot shade.

*One card for each combination of angle of attack and roll.

**Set of cards for all surfaces and a corresponding set for each combination of angle of attack and roll.

5.2 SAMPLE PROBLEM

In order to illustrate the output of this program and how it compares with the version developed under Contract NAS8-20230, a sample case was chosen identical to the one used in Reference 1. The sample case is shown in Figure 8. Referring to Figure 8, the 11 bodies seen define the surface of two intersecting cylinders with a cone frustum on the end of one and a triangular shaped base on the other. Input data for the Sample Problem is shown in Table 1.

5.3 SAMPLE OUTPUT

The example of Section 5.2 was input to the IBM 7094 computer using the program described in this report. Several combinations of angle of attack and roll were chosen, and two altitudes were input to exemplify portions of the program utility. Samples of the data output are shown in Tables 2 and 3.

Table 2 is a sample of the detailed calculations which are performed in determining the overall vehicle coefficients. These bits of datum are useful for checking the accuracy of shadowing on the subshapes and for determining that inputs for the program accurately describe the object in the flow. Free molecule data in this table relate to the first input altitude. Included in the table are the following:

Headings	• For each combination of ALPHA and BETA input to the program, a similar table of data is provided. The angles ALPHA and BETA are printed at the top of each table beside the characters ANGLE OF ATTACK= and ANGLE OF ROLL=, respectively.
NCBS NCS	• Shading flags are outputted for each flow for checking purposes.
BODY NO.	• Type of surface.
PERCENT NOT SHADOWED	• Percent of area not shaded.

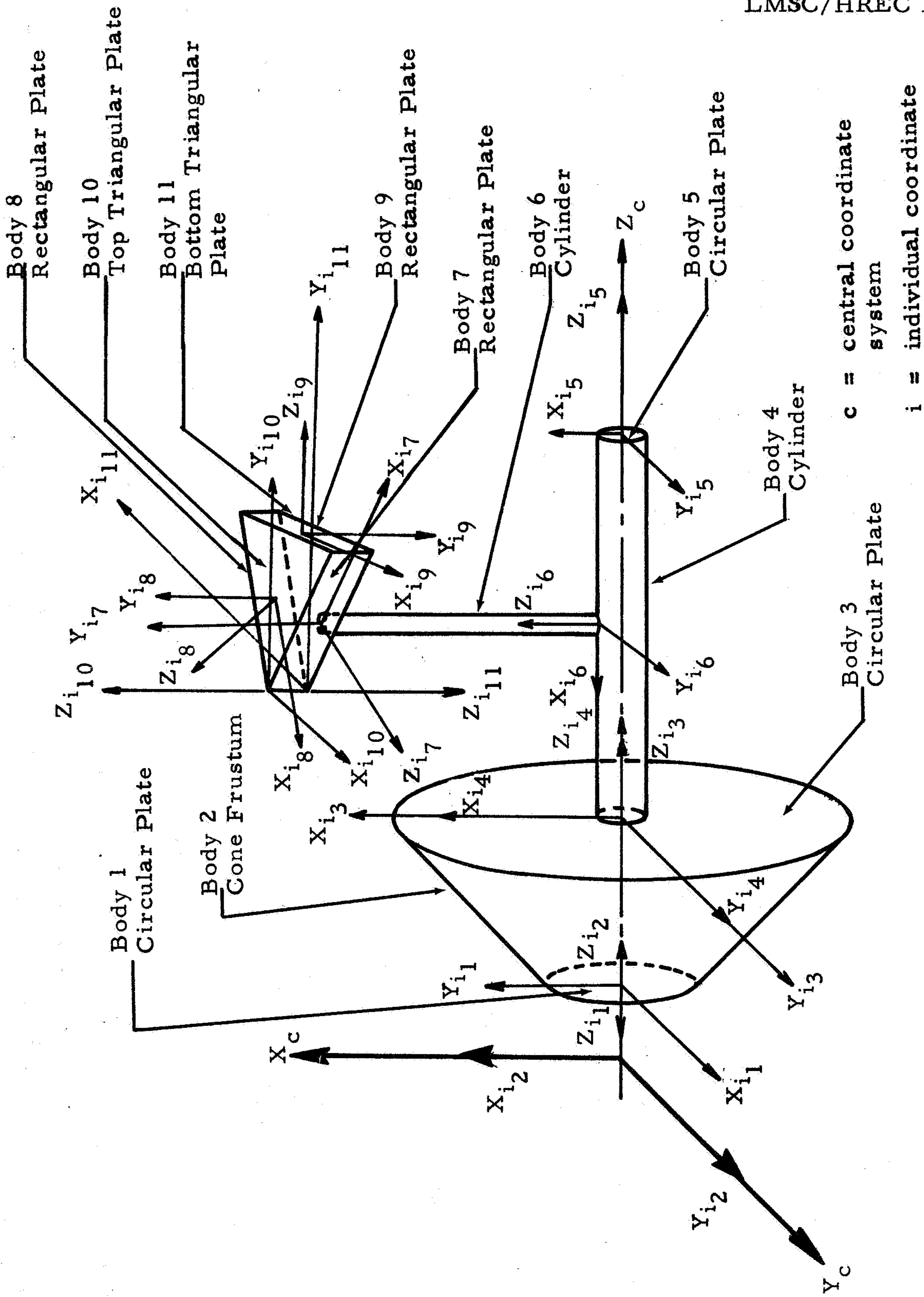


Figure 8 - Sample Problem

Table 1

SAMPLE PROBLEM INPUT DATA

```
                              TEST
 11    8    2         10.         80.          20.
                          6.51                .2465                                16000000.
                        22.559                1.429                                6.413
  1    2    3    3    1      1      +0.         +0.        +10.        +0.         +360.
                           +0.      +0.        +10.        +90.      +180.           +0.
  2    5    5    8    1      1     +45.        +10.        +30.        +0.         +360.
                           +0.      +0.         +0.         +0.        +0.           +0.
  3    2    4   24    1      1      +0.         +3.        +30.        +0.         +360.
                           +0.      +0.      +30.01         +0.        +0.           +0.
  4    4   10    8    1      1      +3.         +0.        +50.        +0.         +360.
                           +0.      +0.      +30.00         +0.        +0.           +0.
  5    2    5    5    1      1      +0.         +0.       +2.95        +0.         +360.
                           +0.      +0.      +80.05         +0.        +0.           +0.
  6    4   10    8    1      1      +1.         +0.      +35.01        +0.         +360.
                           +3.      +0.        +50.         +0.       -90.           +0.
  7    1    8    4    1      1      +0.      -1.975      +1.975     -3.975        +3.975
                          +40.    +2.85        +50.        +90.      +135.           +0.
  8    1    8    4    1      1      +0.      -1.975      +1.975     -3.975        +3.975
                          +40.    -2.85        +50.        +90.      -135.           +0.
  9    1    8    4    1      1      +0.      -1.975      +1.975     -5.655        +5.655
                          +40.      +0.      +52.85        -90.        +0.           +0.
 10    3    6    6    1      1      +0.         +0.      +5.656     -45.00        +45.00
                          +42.      +0.     +47.172        -90.        +0.          +90.
 11    3    6    6    1      1      +0.         +0.      +5.656     -45.00        +45.00
                          +38.      +0.     +47.172        +90.        +0.          +90.
  0.0                 0.0                    6
  1   1 1 1 1   1   1 1   1   1 1       1
  45.0                0.0                    6
  1   1 1 1 1 1 1   1 1 1   1   1   1   1 1
 90.0                 0.0                    3
    1 1 1   1 1     1   1   1   1   1   1 1
 135.0                0.0                    3
        1 1   1     1 1 1   1   1       1
 180.0                0.0                    3
1   1   1 1 1       1 1 1   1     1     1
 225.0                0.0                    7
    1   1 1 1 1     1 1   1   1   1   1
```

Table 1 (Continued)

270.0								0.0						4
	1	1	1		1	1	1	1				1	1	
315.0								0.0						6
				1	1		1	1	1	1	1	1		

Table 2

SAMPLE DATA OUTPUT

```
THE FOLLOWING DATA IS FOR ANGLE OF ATTACK =      0.000          ANGLE OF ROLL =      0.000
NCBS=0 (CANNOT BE SHADED)  NCBS=1 (CAN BE SHADED)  NCS=0 (CANNOT SHADE)  NCS=1 (CAN SHADE)

NCBS(  1)=  -0  NCS(  1)=   1
NCBS(  2)=  -0  NCS(  2)=   1
NCBS(  3)=   1  NCS(  3)=   1
NCBS(  4)=   1  NCS(  4)=  -0
NCBS(  5)=   1  NCS(  5)=  -0
NCBS(  6)=   1  NCS(  6)=   1
NCBS(  7)=  -0  NCS(  7)=   1
NCBS(  8)=  -0  NCS(  8)=   1
NCBS(  9)=   1  NCS(  9)=  -0
NCBS( 10)=  -0  NCS( 10)=  -0
NCBS( 11)=   1  NCS( 11)=  -0
                 BODY NO.      PER CENT NOT SHADOWED    AREA    AREA NOT SHADOWED   NO. OF ELEMENTS   NO. SHADOWED   NO. NOT SHADOWED
          FREE MOLECULAR VALUE                                    CONTINUUM VALUE
BODY   CAT      CNT      CYT      CDT      CMP      CMY      CMR      CAT      CNT      CYT      CMP      CMY      CMR     PCT  ELE.  PJ. AREA

       1----------  2---------------  1.0000             314.16        314.16             9                 0              9
  1  2.1588  0.0000   0.0000   2.1588 -0.0000   0.0000 -0.0000   1.8200   0.0000   0.0000 -0.0000   0.0000 -0.0000  1.0000    9    314.16

       2----------  5---------------  1.0000            3554.31       3554.31            40                 0             40
  2 19.1121  0.0000  -0.0000  19.1121 -0.0000   0.0000 -0.0000   9.1000   0.0000  -0.0000 -0.0000   0.0000 -0.0000  1.0000   40   2513.25
JFLAG(  1)=  1
JFLAG(  2)=  1

       3----------  2---------------  0.0000            2799.16          0.00            96                96              0
  3 19.1121  0.0000  -0.0000  19.1121 -0.0000   0.0000 -0.0000   9.1000   0.0000  -0.0000 -0.0000   0.0000 -0.0000  0.0000   96     0.00
JFLAG(  1)=  1
JFLAG(  2)=  1
JFLAG(  3)=  1

       4----------  4---------------  0.0000             942.48          0.00            80                80              0
  4 19.1121  0.0000  -0.0000  19.1121 -0.0000   0.0000 -0.0000   9.1000   0.0000  -0.0000 -0.0000   0.0000 -0.0000  0.0000   80     0.00
JFLAG(  1)=  1
JFLAG(  2)=  1
JFLAG(  3)=  1
```

(Continued)

Table 2 (Continued)

```
       5----------  2---------------  0.0000             27.34          0.00                25                25           0
  5 19.1121  0.0000 -0.0000 19.1121 -0.0000  0.0000 -0.0000  9.1000  0.0000 -0.0000 -0.0000  0.0000 -0.0000  0.0000   25     0.00
 JFLAG(  1)=  1
 JFLAG(  2)=  1
 JFLAG(  3)=  1
 JFLAG(  7)=  1
 JFLAG(  8)=  1

       6----------  4---------------  0.2000            219.97          22.00               80                72           8
  6 19.2094  0.0000 -0.0000 19.2094  0.1679  0.0000 -0.0000  9.1538  0.0000 -0.0000  0.0929  0.0000 -0.0000  0.2000   80    14.37

       7----------  1---------------  1.0000             31.40          31.40               32                0            32
  7 19.3592  0.0000  0.0034 19.3592  0.4619 -0.0424  0.0165  9.2181  0.0000  0.0643  0.2191 -0.1700  0.1262  1.0000   32    22.20

       8----------  1---------------  1.0000             31.40          31.40               32                0            32
  8 19.5089  0.0000 -0.0000 19.5089  0.7558  0.0000 -0.0000  9.2825  0.0000 -0.0000  0.3453  0.0000  0.0000  1.0000   32    22.20
 JFLAG(  6)=  1
 JFLAG(  7)=  1
 JFLAG(  8)=  1

       9----------  1---------------  0.0000             44.67          0.00                32                32           0
  9 19.5089  0.0000 -0.0000 19.5089  0.7558  0.0000 -0.0000  9.2825  0.0000 -0.0000  0.3453  0.0000  0.0000  0.0000   32     0.00

      10----------  3---------------  1.0000             31.99          31.99               36                0            36
 10 19.5178 -0.0018 -0.0000 19.5178  0.7789  0.0000 -0.0000  9.2825  0.0000 -0.0000  0.3453  0.0000  0.0000  1.0000   36    -0.00
 JFLAG(  6)=  1
 JFLAG(  7)=  1
 JFLAG(  8)=  1

      11----------  3---------------  0.7618             31.99          24.37               36                8            28
 11 19.5245 -0.0004 -0.0000 19.5245  0.7882  0.0000 -0.0000  9.2825  0.0000 -0.0000  0.3453  0.0000  0.0000  0.7618   36    -0.00
                    TØTAL AREA=      8028.875      TØT. NØT SHADØWED=      4039.629      PRØJECTED AREA =      2886.189
```

Table 3

SAMPLE DATA OUTPUT

*******GRAND TØTALS*********** ALTITUDE NUMBER 1

X1	=	80.000	D1	=	20.000	RREF	=	10.000
S	=	6.510	TRTI	=	0.246500	LAMDA	=	16000000.00

ALPHA	BETA	CA	CN	CY	CD	CM	CN	CL	KNUDSEN NØ.
0.0	0.0	19.5245	-0.0004	-0.0000	19.5245	0.7882	0.0000	-0.0000	800000.000
45.0	0.0	10.3388	9.8174	-0.0000	14.2526	-15.5648	0.0000	-0.0000	226274.172
90.0	0.0	0.2119	8.4527	-0.0000	8.4527	-13.2552	0.0000	-0.0000	200000.000
135.0	0.0	-10.7678	9.8404	-0.0000	14.5722	-16.6974	-0.0000	-0.0000	226274.170
180.0	0.0	-20.3374	-0.0003	-0.0000	20.3374	-1.1135	-0.0000	-0.0000	799999.805
225.0	0.0	-10.6548	-9.7469	-0.0000	14.4262	15.6629	-0.0000	0.0000	226274.178
270.0	0.0	0.2119	-8.2075	-0.0000	8.2075	12.6109	0.0000	0.0000	200000.000
315.0	0.0	10.8649	-10.3327	-0.0000	14.9890	17.7054	0.0000	0.0000	226274.170
360.0	0.0	19.5245	-0.0004	-0.0000	19.5245	0.7882	0.0000	-0.0000	800000.000

*******GRAND TØTALS*********** ALTITUDE NUMBER 2

X1	=	80.000	D1	=	20.000	RREF	=	10.000
S	=	22.559	TRTI	=	1.429000	LAMDA	=	6.41

ALPHA	BETA	CA	CN	CY	CD	CM	CN	CL	KNUDSEN NØ.
0.0	0.0	14.1893	-0.0000	-0.0000	14.1893	0.5473	0.0000	0.0000	0.321
45.0	0.0	7.1759	5.1959	-0.0000	8.7482	-10.1703	0.0000	-0.0000	0.091
90.0	0.0	1.4762	3.7631	-0.0000	3.7631	-7.7268	0.0000	-0.0000	0.080
135.0	0.0	-8.4417	2.7651	-0.0000	7.9244	-5.1873	-0.0000	-0.0000	0.091
180.0	0.0	-18.1740	-0.0000	-0.0000	18.1740	-0.9154	-0.0000	-0.0000	0.321
225.0	0.0	-8.2933	-2.7550	-0.0000	7.8124	4.6026	-0.0000	0.0000	0.091
270.0	0.0	1.4762	-3.5855	-0.0000	3.5855	7.2618	0.0000	0.0000	0.080
315.0	0.0	7.4264	-5.4256	-0.0000	9.0877	11.1819	0.0000	0.0000	0.091
360.0	0.0	14.1893	-0.0000	-0.0000	14.1893	0.5473	0.0000	0.0000	0.321

AREA	• Total surface area.
AREA NOT SHADOWED	• Surface area not shaded.
NO. OF ELEMENTS	• Total number of elements on a surface.
NO. SHADOWED	• Number of elements shaded on a surface.
NO. NOT SHADOWED	• Number of elements not shaded on a surface.
BODY	• This column of numbers identifies the rows of data in the table with the surface subshapes of the complex vehicle. For this sample, the numbers 1 - 11 correspond to the surface subshapes as they are labeled in Figure 7.
CAT CNT CYT	• Each column heading signifies that the data below represent the cumulative total of the force coefficient component in the central directions Z_c, X_c and Y_c, respectively, of the "body-fixed" axis system. Values on the left side of the table were computed using existing free molecule flow theory. Values on the right side of the table were computed according to the continuum flow equation.
CDT	• Cumulative force coefficient, drag, in the direction of flow. Each value is computed from components of the three columns which precede it.
CMP CMY CMR	• Each column heading signifies that the data below represent the cumulative total of the aerodynamic moment coefficient components. Using the right-hand rule, the values are positive about the central $-Y_c$, $-X_c$ and $-Z_c$ axes, respectively, of the "body-fixed" axis system. Values on the left side of the table were computed from the free molecule force coefficients. Values on the right side of the table were computed from the continuum force coefficients.

PCT	• This column indicates the percentage of surface area that the program determined was "not shadowed" for each subshape. (Same as PERCENT NOT SHADOWED.)
NO. OF ELE.	• This column indicates the total number of elements that were integrated for each subshape area. (Same as NO. OF ELEMENTS.)
PJ. AREA	• Total surface area projected perpendicular to flow.
JFLAG(ISURFN)	• Flag that determines which surfaces have a possibility of causing shading on the following body.
TOTAL AREA	• Total surface area of all subshapes. Notice that this may not be exact surface area of the complex vehicle, since it would include all the area for two intersecting subshape surfaces.
TOT. NOT SHADOWED	• Total surface area collided with by particles in the flow at the indicated orientation angles for the composite vehicle.
PROJECTED AREA	• Total projected area of all surfaces.

Table 3 is a sample of the total aerodynamic coefficients for the vehicle. Tables are provided for each altitude. This sample illustrates the totals for several combinations of angle of attack and roll, and two altitudes. The flow regimes are indicated by the Knudsen numbers shown for each row of data. Thus, this illustration contains coefficients in the free molecule flow regime and in the transition flow regime for the same vehicle orientation angles. Included in the tables are the following:

Headings	• For each altitude, a similar table is provided. At the beginning of each table, the altitude is indicated by an integer which represents the order in which the altitude control card was input. For this example, ALTITUDE

NUMBER 1 is 850 kilometers and ALTITUDE NUMBER 2 is 100 kilometers.

XI	• Characteristic length of the vehicle from which the Knudsen numbers are computed.
DI	• Characteristic diameter of the vehicle from which the Knudsen numbers are computed.
RREF	• Reference radius of a circular area on which the aerodynamic coefficients are based.
S	• Molecular speed ratio of the mean molecular flow velocity to the circular orbit velocity for the particular input altitude.
TRTI	• Temperature ratio of the reflected-to-incident molecules for the particular input altitude.
LAMDA	• Mean free path of molecules for the particular input altitude.
ALPHA	• Pitch angle of the vehicle in a wind fixed plane.
BETA	• Roll angle of the vehicle in a wind fixed plane.
CA CN CY	• Total aerodynamic force coefficient components in the composite directions Z_c, X_c and $-Y_c$, respectively, of the "body-fixed" axis system.
CD	• Total drag coefficient of the vehicle.
CM CN CL	• Total moment coefficient components. Using the right-hand rule, the values are positive about the composite $-Y_c$, $-X_c$ and $-Z_c$ axes, respectively, of the "body-fixed" axis system.
KNUDSEN NO.	• Knudsen number, which is an index by which the flow regime is determined.

REFERENCES

1. Davis, T. C., A. R. Lake and R. R. Breckenridge, "A Computer Program to Calculate Force and Moment Coefficients on Complex Bodies Formed From Combinations of Simple Subshapes," LMSC/HREC A784522, Lockheed Missiles & Space Company, Huntsville, Ala., August 1967.